

Elementary Derivation of the Dirac Equation. VII

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Z. Naturforsch. **39a**, 142 (1984);
received July 30, 1984

The momentum in Dirac-like electrodynamics

For electrodynamics [1], (5), according to [2], (1) and [2], (5), the following energy balances hold

$$\dot{U} + \operatorname{div} \mathbf{S} = 0, \quad (1)$$

and

$$\dot{\tilde{U}} + \operatorname{div} \tilde{\mathbf{S}} = 0. \quad (2)$$

Whereas (1) reproduces the well-known equations of continuity for the two photon fields described by [1], Eq. (2) represents a completely new relation. From its derivation, as indicated between [2], (4) and [2], (5), it emerges as a peculiarity of Dirac-like electrodynamics [1], (5) or [1], (8), respectively.

Explicit evaluation of (2) yields

$$\begin{aligned} \dot{\tilde{U}} = & \varepsilon (E_1^{\operatorname{Im}} E_2^{\operatorname{Re}} - E_2^{\operatorname{Im}} E_1^{\operatorname{Re}}) \\ & + \mu (H_1^{\operatorname{Im}} H_2^{\operatorname{Re}} - H_2^{\operatorname{Im}} H_1^{\operatorname{Re}}), \end{aligned} \quad (3)$$

and

$$\begin{aligned} \frac{1}{c} \dot{\tilde{\mathbf{S}}} = & \mathbf{H}^{\operatorname{Re}} E_3^{\operatorname{Im}} - \mathbf{H}^{\operatorname{Im}} E_3^{\operatorname{Re}} + \mathbf{E}^{\operatorname{Im}} H_3^{\operatorname{Re}} - \mathbf{E}^{\operatorname{Re}} H_3^{\operatorname{Im}} \\ & - \mathbf{e}_3 [(E^{\operatorname{Im}} \cdot \mathbf{H}^{\operatorname{Re}}) - (E^{\operatorname{Re}} \cdot \mathbf{H}^{\operatorname{Im}})]. \end{aligned} \quad (4)$$

By introducing a vector

$$\begin{aligned} \mathbf{V} = & \varepsilon (\mathbf{E}^{\operatorname{Im}} \times \mathbf{E}^{\operatorname{Re}}) + \mu (\mathbf{H}^{\operatorname{Im}} \times \mathbf{H}^{\operatorname{Re}}) \quad \text{with} \\ (\mathbf{V} \cdot \mathbf{e}_3) = & \dot{\tilde{U}}, \end{aligned} \quad (5)$$

and a tensor

$$\begin{aligned} \frac{1}{c} \mathbf{T} = & \mathbf{H}^{\operatorname{Re}} \mathbf{E}^{\operatorname{Im}} - \mathbf{H}^{\operatorname{Im}} \mathbf{E}^{\operatorname{Re}} + \mathbf{E}^{\operatorname{Im}} \mathbf{H}^{\operatorname{Re}} - \mathbf{E}^{\operatorname{Re}} \mathbf{H}^{\operatorname{Im}} \\ & - \mathbf{1} [(E^{\operatorname{Im}} \cdot \mathbf{H}^{\operatorname{Re}}) - (E^{\operatorname{Re}} \cdot \mathbf{H}^{\operatorname{Im}})] \\ \text{with } (\mathbf{T} \cdot \mathbf{e}_3) = & \tilde{\mathbf{S}}, \end{aligned} \quad (6)$$

we can write (2) as

$$(\dot{\mathbf{V}} + \operatorname{div} \mathbf{T}) \cdot \mathbf{e}_3 = 0. \quad (7)$$

In contrast to the energy balance (1), where the time derivative involves a scalar quantity, i.e. the electromagnetic energy density, in (7), or (2) respectively, the time derivative of a vector appears. This vector is constructed from the vector products

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of the electric and magnetic parts of the two photons and has the direction of the common axis of revolution of both photon wave fields. Most obvious assumption: (7) represents a momentum balance, (5) denotes the angular and spin momentum of the two-photon system, and (6) the corresponding flow tensor. The balance (7) gives an immediate basis to the model concepts below [3], (12). Equation (7) evidently is valid for an electromagnetic momentum vortex with a special direction: its axis points into the 3-axis. – The balance will be now established in a completely general way with the help of Dirac-like electrodynamics [1], (5).

First one computes the divergence of the flow tensor from (6):

$$\begin{aligned} \frac{1}{c} \operatorname{div} \mathbf{T} = & (\operatorname{div} \mathbf{H}^{\operatorname{Re}}) \mathbf{E}^{\operatorname{Im}} + (\mathbf{H}^{\operatorname{Re}} \cdot \operatorname{grad}) \mathbf{E}^{\operatorname{Im}} \\ & - (\operatorname{div} \mathbf{H}^{\operatorname{Im}}) \mathbf{E}^{\operatorname{Re}} - (\mathbf{H}^{\operatorname{Im}} \cdot \operatorname{grad}) \mathbf{E}^{\operatorname{Re}} \\ & + (\operatorname{div} \mathbf{E}^{\operatorname{Im}}) \mathbf{H}^{\operatorname{Re}} + (\mathbf{E}^{\operatorname{Im}} \cdot \operatorname{grad}) \mathbf{H}^{\operatorname{Re}} \\ & - (\operatorname{div} \mathbf{E}^{\operatorname{Re}}) \mathbf{H}^{\operatorname{Im}} - (\mathbf{E}^{\operatorname{Re}} \cdot \operatorname{grad}) \mathbf{H}^{\operatorname{Im}} \\ & - \operatorname{grad} [(E^{\operatorname{Im}} \cdot \mathbf{H}^{\operatorname{Re}}) - (E^{\operatorname{Re}} \cdot \mathbf{H}^{\operatorname{Im}})]. \end{aligned} \quad (8)$$

Because of the second line of [1], (5) the four terms containing divergences drop out. With help of the vector relation

$$\begin{aligned} \operatorname{grad} (\mathbf{a} \cdot \mathbf{b}) = & (\mathbf{b} \cdot \operatorname{grad}) \mathbf{a} + (\mathbf{a} \cdot \operatorname{grad}) \mathbf{b} \\ & + (\mathbf{a} \times \operatorname{rot} \mathbf{b}) + (\mathbf{b} \times \operatorname{rot} \mathbf{a}) \end{aligned} \quad (9)$$

one can see that four more terms in (8) cancel with the last one. Thus the following relation remains:

$$\begin{aligned} \frac{1}{c} \operatorname{div} \mathbf{T} + & (\mathbf{E}^{\operatorname{Im}} \times \operatorname{rot} \mathbf{H}^{\operatorname{Re}}) + (\mathbf{H}^{\operatorname{Re}} \times \operatorname{rot} \mathbf{E}^{\operatorname{Im}}) \\ & - (\mathbf{E}^{\operatorname{Re}} \times \operatorname{rot} \mathbf{H}^{\operatorname{Im}}) - (\mathbf{H}^{\operatorname{Im}} \times \operatorname{rot} \mathbf{E}^{\operatorname{Re}}) = 0. \end{aligned} \quad (10)$$

Here the last four terms immediately yield the momentum rate (5) because of the first line of [1], (5), so that the momentum balance (7) follows in complete generality as

$$\dot{\mathbf{V}} + \operatorname{div} \mathbf{T} = 0. \quad (11)$$

The direct derivation of the momentum balance (11) from the two photon electrodynamics once again shows the Dirac structure to be a quite central system-immanence of electrodynamics.

- [1] H. Sallhofer, Z. Naturforsch. **33a**, 1378 (1978).
- [2] H. Sallhofer, Z. Naturforsch. **34a**, 1145 (1979).
- [3] H. Sallhofer, Z. Naturforsch. **35a**, 995 (1980).

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